



AP[®] Calculus AB 2011 Free-Response Questions

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2011 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

1. For $0 \leq t \leq 6$, a particle is moving along the x -axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2 \sin(e^{t/4}) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2} e^{t/4} \cos(e^{t/4})$ and $x(0) = 2$.
- (a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.
- (b) Find the average velocity of the particle for the time period $0 \leq t \leq 6$.
- (c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.
- (d) For $0 \leq t \leq 6$, the particle changes direction exactly once. Find the position of the particle at that time.

WRITE ALL WORK IN THE EXAM BOOKLET.

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t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

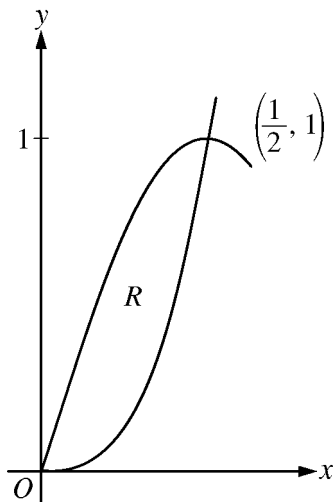
2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \leq t \leq 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.
- (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.
- (b) Using correct units, explain the meaning of $\frac{1}{10} \int_0^{10} H(t) dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10} \int_0^{10} H(t) dt$.
- (c) Evaluate $\int_0^{10} H'(t) dt$. Using correct units, explain the meaning of the expression in the context of this problem.
- (d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

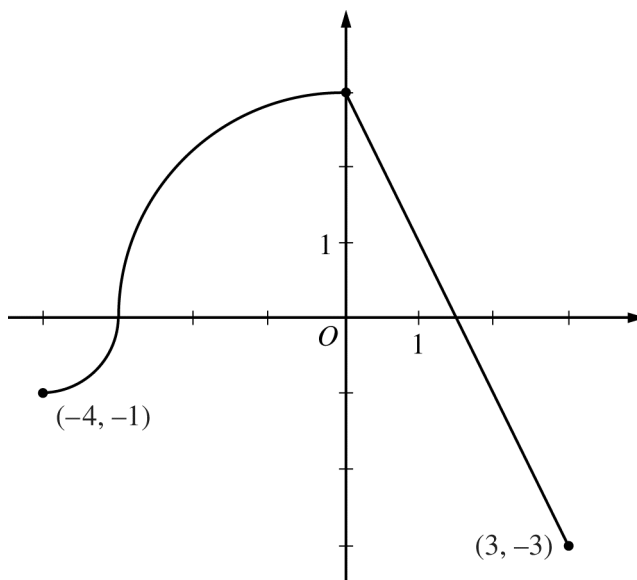
CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



3. Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.
- Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.
 - Find the area of R .
 - Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

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Graph of f

4. The continuous function f is defined on the interval $-4 \leq x \leq 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_0^x f(t) dt$.
- Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.
 - Determine the x -coordinate of the point at which g has an absolute maximum on the interval $-4 \leq x \leq 3$. Justify your answer.
 - Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.
 - Find the average rate of change of f on the interval $-4 \leq x \leq 3$. There is no point c , $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

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5. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.
- (a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).
- (b) Find $\frac{d^2W}{dt^2}$ in terms of W . Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.
- (c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.
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6. Let f be a function defined by $f(x) = \begin{cases} 1 - 2 \sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

- (a) Show that f is continuous at $x = 0$.
- (b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.
- (c) Find the average value of f on the interval $[-1, 1]$.
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END OF EXAM